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SUGGESTIONS ON THE THEORY OF VALUE.[1]

THE endeavor is made in this paper to show that there are two classes of commodities, first-hand and second-hand commodities. For the price of each a special formula is given in place of the general formula of demand and supply.

I.

Many forces are at work on the economic condition of society, but the crops are easily the first in importance. Their influence is so all-pervading that just because they themselves are separated from each other by the interval of a year the consumption and production of any commodity in any one year is clearly distinguished from (but of course not equal to) the consumption and production of the year previous and the year following it. In consequence of this demarcation two classes of commodities can be distinguished:[2]—

Commodities which are produced for the current period of consumption may be called first-hand commodities.

Commodities which were produced for the consumption of a previous period of consumption and such commodities of the current period as have already gone into the hands of consumers may be called second-hand commodities.

Some well-known examples of second-hand commodities are old statues, paintings, books, wines, coins, and furniture. But a hoe manufactured last year, even if still in the hands of the manufacturer, and a hat which a consumer has just bought, are second-hand commodities just as much as the helmet of a mediæval knight.

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[1] The writer of this paper, with an obliging disposition to economize space in our columns, has expressed himself with the utmost brevity. His meaning will be none the less clear to those who are conversant with the theoretic questions on which he touches.—THE EDITOR.

[2] The germ of this distinction is to be found in every part of the theory of price, but only in the theory of interest has its importance been clearly recognized.

It follows from these definitions that a commodity the production of which has ceased is a second-hand commodity. A demand which must be satisfied from the stock on hand and entirely excludes a stock to be produced must be considered a demand for a second-hand commodity, for in regard to such demand the production of the commodity has ceased.

Popularly, two periods are distinguished in the life of a second-hand commodity. In the first period its properties are identical with those of first-hand commodities. They enter the second period when time has imparted to them some property which differentiates them from the first-hand commodity. Some second-hand commodities, like diamonds and gold, remain always in the first period; while others, like wearing apparel and wines, enter the second period almost as soon as they emerge as second-hand commodities.

The distinction of these periods is of no importance in the problem of price; but it can be truly said that first-hand commodities are produced by man, while second-hand commodities are created by time.

The sale of a first-hand commodity is a transaction which takes place between a producer and a consumer.

The sale of a second-hand commodity is a transaction which takes place between two consumers.

As mentioned above, a hoe manufactured last year and still in the hands of the manufacturer is a second-hand commodity. A sale of this hoe would apparently be a transaction between a producer and a consumer. But it must be remembered that here, as in every other investigation of price, it is assumed (even if not always explicitly stated) that the product of a period goes into the hands of consumers within the same period. A producer who for any reason whatsoever retains a part of the production of the current period beyond this period must to that extent be considered a consumer.

II.

It is customary to call the utility of the last unit of a commodity owned the marginal utility. The utility which an additional unit would have to the individual may be called the intensity of desire. In consequence of the law of diminishing utility the intensity of desire is smaller than the marginal utility, and they may differ largely.

Marginal utility measured in money is apparently the consumer's minimum offer. For egoism will prompt the consumer to sell the marginal unit whenever he can get for it more than its marginal utility expressed in money, because he would secure a surplus, and not to sell it for less than this, because he would incur a loss.

Similarly, the intensity of desire expressed in money is the consumer's maximum bid; that is, the highest amount at which he is willing to buy an additional unit.

Between the different consumers of one and the same second-hand commodity exists commercial competition; that is, there exists between them a rivalry based on the knowledge of each other's maximum bids and minimum offers. As long as the maximum bid of any consumer is greater than the minimum offer of another consumer, sales will be made. Ultimately, the stock will be found distributed according to the following formula:—

The whole stock of a second-hand commodity is distributed among the different consumers so that the highest maximum bid is still lower than the lowest minimum offer.

This theorem is self-evident in the form given; but, as the maximum bids and minimum offers are functions of the stock owned, the price can, with the aid of it, be determined numerically.

It is applicable whether the commodity is "capable of indefinite subdivision"[1] or not. If it is, the marginal utility will become equal to the intensity of desire, and the above theorem then says that the commodity will be distributed among the different consumers so that their marginal utilities measured in money, or, what is the same in this case, their intensities of desire, will be equal to each other. This is the meaning of Jevons's equations, and really the whole of his theory.

The above formula is a counterpart to Gossen's law. The latter states that a (single) consumer will invest his means in several commodities, so that his maximum bids for all the different commodities will be equal. The above formula says that a single commodity will be distributed among the several consumers so that their maximum bids for it will all be equal.

The theorem given is more symmetrical than the formula of demand and supply. The latter induces a belief as if there existed a quantity demanded, distinct and different from the quantity supplied. As a matter of fact, the whole existing quantity of a second-hand commodity is demanded as well as supplied (in the ordinary meaning of these terms). The problem is only to ascertain its division among the different consumers, and the above formula gives the solution.

Most economic phenomena on price show themselves most clearly in the dealings on stock exchanges, and there the formula of demand and supply is never used. The greatest maximum bid and the lowest minimum offer are well recognized as the factors determining the price, and together they form what is called the quotation.

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[1] W. S. Jevons, *Theory of Political Economy*, 3d edition, p. 111. See also p. 100. He treats only the special case of commodities capable of indefinite subdivision. Naturally, his equations are not applicable to the more general case of indivisible commodities, and he enumerates himself a great number of instances in which they fail.

The theorem also determines the distribution of an individual's possession of a commodity into two stocks, capital and private fortune. Each of these stocks has its own maximum bid and minimum offer. For instance, a dealer in shoes distributes his shoes between his business and his private fortune. Occasionally, the maximum bid of the latter is greater than the minimum offer of the former. Some shoes will be transferred from his business to his private fortune, till again the latter's maximum bid becomes smaller than the minimum offer of the former.

III.

The total expense of producing a first-hand commodity may be divided into a general expense and a producing expense. As items constituting the general expense, there may be enumerated cost of management, rents, insurance, depreciation of buildings and machinery, cost of advertising, and interest on the money invested in buildings and machinery.

But in some operations the general expense comprises many more items. For instance, all expenses of a theatrical performance, the wages of the actors included, are general expenses. Practically all expenses of a steamer plying between two points are general expenses. In this case even the expense for fuel and the wages of all people employed are general expenses. Practically the whole expense of running a train regularly between two points is general expense.[1] The expense for fuel, the wages of the engineer, fireman, and brakemen employed on that train, are general expenses. As almost the whole passenger traffic is carried on by trains running regularly between two points, almost all expenses of passenger transportation must be considered general expense.

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[1] Many people do not hesitate to accept theatre and railroad passes, precisely because their producing expense is zero. This, together with the theorem that the general expense does not enter into the determination of price, also explains why passenger rates fluctuate so much less than freight rates.

There is no production the expense of which consists only of producing expense. The general expense is never zero.

Neither the general expense per unit nor the producing expense is the same in two factories. The differences in regard to the first are much smaller than is commonly believed. Theoretically, the larger plant should have a smaller general expense per unit of capacity than the smaller plant; for most of the items enumerated above as comprising the general expense grow smaller per unit, if the capacity increases. But against this must be set the practical necessity in larger plants of making the supervision more thorough and costly. It entails many expenses which are entirely missing in the smaller plant. No general theoretical statement can therefore be made, but experience seems to show that the general expense per unit of capacity has several minima.

There is a certain capacity which just permits the manufacturer a satisfactory personal supervision. This furnishes the lowest minimum. Beyond it he needs the help of managers and other assistants, but the business remains centralized in one place, in one head. Such an organization gives rise to another minimum. Beyond that the organization becomes more complicated: it must be decentralized; and there is a certain capacity which with such an organization makes the general expense per unit a minimum.

In regard to the producing expense, the factory with the larger tools has a great advantage; for an increase in the size of the tools brings about a decrease in the producing expense per unit of capacity.

IV.

First-hand commodities may be divided into two classes, specialties and staples.

A specialty is a commodity which is produced by one producer only. This comprises production under patents and trade-marks, and all cases in which the brand partakes of the nature of a trade-mark.

A staple is a commodity which is produced by two or more producers.

The net income of the manufacturer equals the difference between the gross income and total expense. Each of these three quantities depends on the amount produced. The total expense always increases with the amount produced, but the relation between the latter and the two other quantities is more complex.

The gross income equals the quantity sold multiplied by the price. The quantity produced must in the long run equal the quantity sold. The latter is subject to the law of demand, or, to use the term employed by business men, to the law of consumption. The higher the price, the less will be consumed of a commodity. It follows from this law that the gross income has a maximum. For at the price zero the quantity that can possibly be sold is at its maximum, but the gross income is zero. At a price which is so high that it stops all sales the gross income is again zero. Between zero and the latter price there must be a price which makes the gross income a maximum.

The net income has not necessarily a maximum. As a matter of fact, it may be steadily negative; that is, there may never be a real net income, but a loss. Several cases are possible. If, for every capacity that may be chosen, the cost of production is greater than the price at which this quantity would be consumed, the manufacture is unprofitable. In the second case the manufacture yields a net income within certain limits of the capacity. A factory with a capacity of less than the lower limit is said to fail for lack of capital. The factory with a capacity greater than the upper limit may be made profitable by what is called a reduction of capital, but what really is a reduction of capacity. In the third case there is no lower limit, and every factory up to a certain capacity yields a net income.

The manufacturer of a specialty must start from the law of consumption as a predetermined fact. The point which he must decide is the capacity of the plant he desires to erect. After it is ascertained that the specialty comes within the second or third case, he will select that capacity which will make the net income a maximum. It is clear that there is a maximum. For at the capacity zero the net income is negative; that is, there is a loss (equalling the total general expense). Under the supposition made there will be a capacity, beginning from which there will be really a positive net income. Beyond a certain capacity (the upper limit of the second and third case) the net income will again become negative. But, if a quantity is first negative, then positive, and afterwards again negative, it has a maximum.

The capacity which gives the maximum net income is smaller than the capacity which gives the maximum gross income. This may be seen as follows: The net income equals the difference of gross income and expense. In the immediate neighborhood of the capacity giving the maximum gross income, small changes of the former leave the latter practically unchanged, for this is a characteristic property of a maximum. If, then, the capacity increases from a little below that capacity to a little above it, the minuend in the above difference remains practically unchanged, but the subtrahend, the expense, and consequently the net income, continually diminish in that interval. Therefore, the capacity giving maximum gross income cannot be the one giving maximum net income. Beyond that capacity the minuend continually decreases, the subtrahend continually increases, so that the net income continually decreases, and its maximum cannot possibly be in this region. But, as there actually is a maximum, it must be reached before that of the gross income.

There is only one exception to this theorem. In those productions in which all expenses are general expenses, the capacity giving maximum gross income also gives the maximum net income.

V.

The price of a staple will be investigated first under the condition that the cost of production of all producers is the same. The solution of this problem is familiar; but, in order to bring out more fully that the price depends on the relation of producing capacity to consumption, it will be given in the form of the following two theorems:—

If the producing capacity is smaller than the amount which would be consumed at a price equalling cost of production, the price will be raised beyond that cost to such a figure as will make the amount required for consumption at that figure just equal the producing capacity.

If the producing capacity is greater than the amount which would be consumed at a price equalling cost of production, the latter will become the price of the staple.

The law of consumption has been assumed to be a predetermined factor, known to the manufacturer. But, as a matter of fact, the consumption of many staples is subject to frequent changes. The manufacturer judges the latter from the increase or decrease of his sales. If his daily sales or orders become larger than his daily capacity, he will raise the price and thus reduce consumption. He will raise it to such a figure as will just permit him to sell his full capacity. He constantly compares the rate of his sales or orders—*i.e.*, the rate of consumption—with his producing capacity.

If the latter is greater than the amount that would be consumed at a price equalling the cost of production, the factories cannot possibly produce their full capacity, for they cannot dispose of it even at cost of production, and they certainly will not manufacture in order to sell below it. Consequently, each factory will produce only a part of its capacity. The manufacturer will now notice what a large share the general expense has in the total expense, and how quickly this share grows, if the factory produces less than the full capacity. He accordingly reduces the price (if it is still high) in order to increase his sales, and the competition with the other manufacturers reduces it to the cost of production.

The rate of consumption in its relation to the producing capacity thus appears as the factor determining the price. The latter will equal the cost of production and will remain equal to it, even when the ratio changes, if it only remains as stated; that is, as long as the consumption is smaller than the capacity. This is seen clearest during so-called bad times, when the price remains equal to the cost, and unchanged even after all indications point to an increased consumption.

If the costs of production of the different manufacturers differ, a producer with a lower cost will underbid a competitor with the higher cost till his capacity is fully employed. The price will equal the highest cost of production which permits the full capacity existing at that cost and below it to go into consumption. If this latter amount is greater than the whole capacity, the price will be raised to such a figure as will again make the consumption equal to the capacity.

LUDWIG KOTANY.

ST. LOUIS, MO.

NOTE ON THE MATHEMATICAL THEORY OF COMPETITION.

Cournot's treatment of competition[1] is based on the assumption that each producer considers his rival's output as constant. This assumption is quite inadmissible, because the quantities sold by them are obviously functions of each other.

Suppose, then, there are two producers. Let their respective sales be s_1 and s_2, their expense functions $\xi_1(s_1)$ and $\xi_2(s_2)$, their marginal costs

$$e_1 = \frac{d\,\xi_1(s_1)}{ds_1} \text{ and } e_2 = \frac{d\,\xi_2(s_2)}{ds_2}.$$

The law of demand or consumption will be given in the form

$$(1)\ \ldots\ p = f(s),$$

where

$$(2)\ \ldots\ s = s_1 + s_2.$$

The net incomes are

$$n_1 = s_1 p - \xi_1(s_1),$$
$$n_2 = s_2 p - \xi_2(s_2).$$

They will be maxima if

$$p + s_1 \frac{dp}{ds}\left(1 + \frac{ds_2}{ds_1}\right) - e_1 = 0,$$

$$p + s_2 \frac{dp}{ds}\left(1 + \frac{ds_1}{ds_2}\right) - e_2 = 0.$$

Eliminating $\frac{ds_2}{ds_1}$ from these two equations gives

$$(3)\ \ldots\ (p - e_1)(p - e_2) + \frac{dp}{ds}[s_1(p - e_2) + s_2(p - e_1)] = 0.$$

In the case of manufacturers e_1 and e_2 are constants. Suppose first they are equal, so that $e_1 = e_2 = e$. Then equation (3) can be written in the form

$$(4)\ \ldots\ (p - e)\left(p - e + s\frac{dp}{ds}\right) = 0.$$

This equation gives rise to two solutions according as the first or second factor vanishes. The first factor vanishes, if

$$(5)\ \ldots\ p = e.$$

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[1] Cournot's *Researches* (translated by Bacon), chaps. vii. and viii.

This is the case of competition. The price equals the cost of production, and s will be found from equation (1); but the equations are insufficient to determine s_1 and s_2,—a result fully borne out by experience.

The second solution is based on the equation

$$(6) \ldots\ p - e + s\frac{dp}{ds} = 0.$$

This is the case of a trust. The equation shows that the trust price of a staple is determined from exactly the same equation as determines the price of a specialty. The individual sales s_1 and s_2 can again not be determined from these equations, but from some extraneous considerations,—a result quite in keeping with experience.

The two solutions show that the producers of a staple whose costs are equal must adopt either the cost price or the specialty price. The close relation thus established between competition and trust as two solutions of one and the same equation, and the fact that this equation has only these two solutions, and no more, must be considered an important theorem in itself.

There still remains the investigation of equation (3) in the case that the marginal costs are unequal constants.

Suppose, then, e_1 and e_2 are constants and $e_1 > e_2$. The left side of equation (3) is a positive quantity for $p = 0$; it is still positive for $p = e_2$, but negative for $p = e_1$, and again positive (and infinitely great) for $p = \infty$. From this it follows that one solution of the equation lies between e_1 and e_2, and the other beyond e_1. But the first solution is not acceptable to the producer with the cost e_1, because he cannot remain in business if the price is below his cost (see Article V). The only economically possible solution is, therefore, the second. This means that competition cannot exist between manufacturers of different costs, but that a trust agreement may be maintained between them.

These results are true only if the limits of the respective producing capacities are not reached, as Cournot has already pointed out. If they are reached, competition may exist. The price, then, is determined from the demand function $p = f(c)$, where c is the sum of the capacities of the two producers. But this is again the same price as is got for a specialty, if the capacity of the monopolist is reached.